

Pre Calculus Honors

Unit 4 Unit Circle Trigonometry and Triangles Test Prep - DUE _____

You should already have a personal study guide covering the following general topics

- What are inverses (graphically, function wise, and table-based) and how can they be created?
- What are the characteristics of even/odd functions?
- How can functions be manipulated algebraically using properties?
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1. To prepare for your test **add content to your previous personal study guide**. Use the index below to pull out important topics from Unit 4

4.1

- A. Radians and Degrees
- B. Sketching angles accurately
Vocabulary (coterminal, reference, terminal side vs. initial side, standard position, positive vs. negative angles)

4.2

- A. Trig Functions on Right Triangles
- B. Special Right Triangles (30-60-90 and 45-45-90)
- C. Right triangle trig word problems
- D. Angles through a point

4.3

- A. **THE UNIT CIRCLE** (create it, define it, know it's values)
- B. Unit Circle expressions and periodic properties, even/oddness

4.4

- A. Inverse Trig Functions
- B. Nested and Algebraic Inverse Trig functions
- C. Trig Word Problems requiring inverse functions (missing angle)

4.5

- A. Law of Sines + the Ambiguous Case
- B. Law of Cosines
- C. Word Problem Application of Law of (Co)Sines

4.6

- A. Area of Triangles + Application

2. **Complete at least twenty problems** from the bank on the back of this study guide. Focus on the areas you need to practice, but make sure you look over all types of problems/topics. *The answer key is attached. I will be checking your work for the twenty problems.* This guide is meant to be a general, *basic* review of the topics covered in Unit 4

3. **Review** your quizzes, homework, classwork practice, and warm-ups. Practice specifically with the problems you missed!

Your study guide (3 points) and the practice problems (2 points) are homework grades.

Your test is on _____. You are welcome to attend a tutorial or stop by during lunch (any day but Tuesday) with specific questions or areas to review. Please let Mrs. Pike know in advance that you are coming!

Unit 4 Lesson 1 #2-17

1. Find the radian measure of the central angle of a circle of radius 6 centimeters that intercepts an arc of length 27 centimeters.

In Exercises 2-4, convert each angle in degrees to radians. Express your answer as a multiple of π .

2. 15° 3. 120° 4. 315°

In Exercises 5-7, convert each angle in radians to degrees.

5. $\frac{5\pi}{3}$ 6. $\frac{7\pi}{5}$ 7. $-\frac{5\pi}{6}$

In Exercises 8-12, draw each angle in standard position.

8. $\frac{5\pi}{6}$ 9. $-\frac{2\pi}{3}$ 10. $\frac{8\pi}{3}$
11. 190° 12. -135°

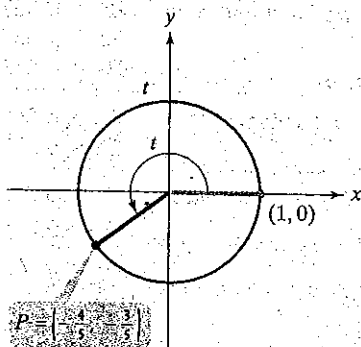
In Exercises 13-17, find a positive angle less than 360° or 2π that is coterminal with the given angle.

13. 400° 14. -445° 15. $\frac{13\pi}{4}$
16. $\frac{31\pi}{6}$ 17. $-\frac{8\pi}{3}$

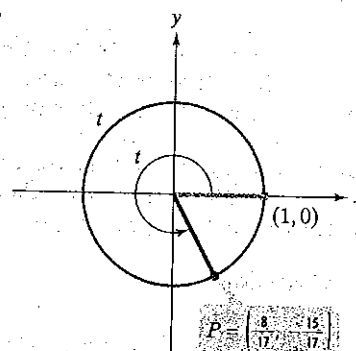
Unit 4 Lesson 3 #21-27

In Exercises 21-22, a point $P(x, y)$ is shown on the unit circle corresponding to a real number t . Find the values of the trigonometric functions at t .

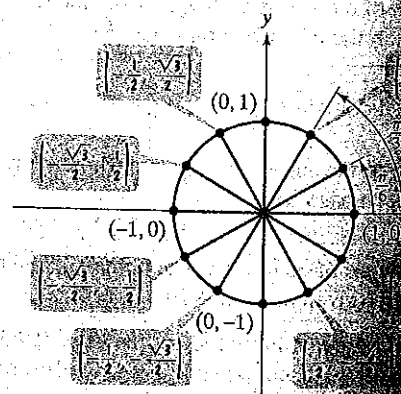
21.



22.



In Exercises 23-26, use the figure shown to find the value of the trigonometric function at the indicated real number t . If the expression is undefined.



23. $\sec \frac{5\pi}{6}$

24. \tan

25. $\sec \frac{\pi}{2}$

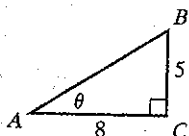
26. \cos

27. If $\sin t = \frac{2\sqrt{7}}{7}$, $0 \leq t < \frac{\pi}{2}$, use the remaining trigonometric functions.

Unit 4 Lesson 2 #31-43

Unit 4 Lesson 2 #45-67

Use the triangle to find each of the six trigonometric functions of θ .



Exercises 32-35, find the exact value of each expression. Do not use a calculator.

$$\sin \frac{\pi}{6} + \tan^2 \frac{\pi}{3}$$

$$33. \cos^2 \frac{\pi}{4} - \tan^2 \frac{\pi}{4}$$

$$\csc \frac{\pi}{5} - \tan^2 \frac{\pi}{5}$$

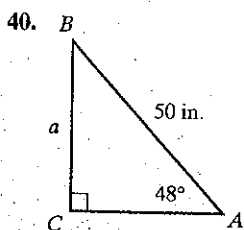
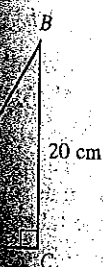
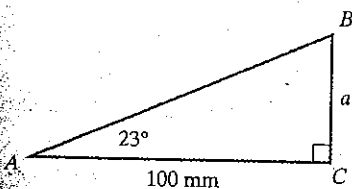
$$35. \cos \frac{2\pi}{9} \sec \frac{2\pi}{9}$$

Exercises 36-37, find a cofunction with the same value as the expression.

$$\sin 40^\circ$$

$$37. \cos \frac{\pi}{2}$$

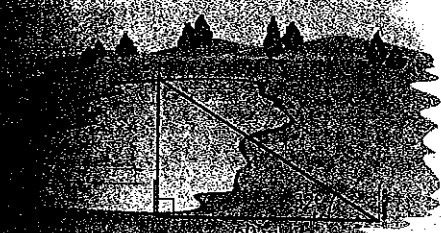
Exercises 38-40, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to nearest whole number.



and θ is acute, find $\tan\left(\frac{\pi}{2} - \theta\right)$.

limbs for a half mile up a slope whose inclination is 30° . How many feet of altitude, to the nearest foot, does the climber gain?

To measure the distance across a lake, a surveyor took the measurements in the figure shown. What is the distance across the lake? Round to the nearest meter.



In Exercises 45-46, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ , or state that the function is undefined.

$$45. (-1, -5)$$

$$46. (0, -1)$$

In Exercises 47-48, let θ be an angle in standard position. Name the quadrant in which θ lies.

$$47. \tan \theta > 0 \text{ and } \sec \theta > 0$$

$$48. \tan \theta > 0 \text{ and } \cos \theta < 0$$

In Exercises 49-51, find the exact value of each of the remaining trigonometric functions of θ .

$$49. \cos \theta = \frac{2}{5}, \sin \theta < 0$$

$$50. \tan \theta = -\frac{1}{3}, \sin \theta > 0$$

$$51. \cot \theta = 3, \cos \theta < 0$$

In Exercises 52-56, find the reference angle for each angle.

$$52. 265^\circ$$

$$53. \frac{5\pi}{8}$$

$$54. -410^\circ$$

$$55. \frac{17\pi}{6}$$

$$56. -\frac{11\pi}{3}$$

In Exercises 57-67, find the exact value of each expression. Do not use a calculator.

$$57. \sin 240^\circ$$

$$58. \tan 120^\circ$$

$$59. \sec \frac{7\pi}{4}$$

$$60. \cos \frac{11\pi}{6}$$

$$61. \cot(-210^\circ)$$

$$62. \csc\left(-\frac{2\pi}{3}\right)$$

$$63. \sin\left(-\frac{\pi}{3}\right)$$

$$64. \sin 495^\circ$$

$$65. \tan \frac{13\pi}{4}$$

$$66. \sin \frac{22\pi}{3}$$

$$67. \cos\left(-\frac{35\pi}{6}\right)$$

Unit 4 Lesson 4 # 94 - 114

In Exercises 94-112, find the exact value of each expression. Do not use a calculator.

94. $\sin^{-1} 1$

95. $\cos^{-1} 1$

96. $\tan^{-1} 1$

97. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

98. $\cos^{-1}\left(-\frac{1}{2}\right)$

99. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

100. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

101. $\sin(\cos^{-1} 0)$

102. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

103. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

104. $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$

105. $\cos\left(\tan^{-1}\frac{3}{4}\right)$

106. $\sin\left(\cos^{-1}\frac{3}{5}\right)$

107. $\tan\left[\sin^{-1}\left(\frac{2}{5}\right)\right]$

108. $\tan\left[\cos^{-1}\left(-\frac{4}{5}\right)\right]$

109. $\sin\left[\tan^{-1}\left(\frac{1}{2}\right)\right]$

110. $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$

111. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

112. $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$

In Exercises 113-114, use a right triangle to write each as an algebraic expression. Assume that x is positive and the inverse trigonometric function is defined for the expression.

113. $\cos\left(\tan^{-1}\frac{x}{2}\right)$

114. $\sec\left(\sin^{-1}\frac{x}{2}\right)$

Unit 4 Lesson 5 # 2-16, 17-21

1-12, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree. If no triangle exists, write "no triangle." If two triangles exist, solve each triangle.

1. $B = 55^\circ, a = 12$ 2. $B = 107^\circ, C = 30^\circ, c = 126$

3. $a = 17, c = 12$ 4. $a = 117, b = 66, c = 142$

5. $B = 25^\circ, c = 68$ 6. $A = 39^\circ, a = 20, b = 26$

7. $a = 3, c = 1$

8. $\angle A = 100^\circ, b = 11.2, c = 48.2$

9. $a = 26.1, b = 40.2, c = 36.5$

10. $A = 40^\circ, a = 6, b = 4$ 11. $B = 37^\circ, a = 12.4, b = 8.7$

12. $A = 23^\circ, a = 54.3, b = 22.1$

In Exercises 13-16, find the area of the triangle having the given measurements. Round to the nearest square unit.

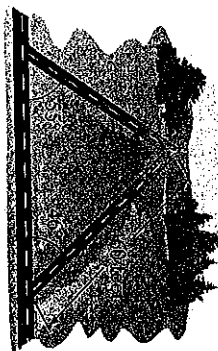
13. $C = 42^\circ, a = 4$ feet, $b = 6$ feet

14. $A = 22^\circ, b = 4$ feet, $c = 5$ feet

15. $a = 2$ meters, $b = 4$ meters, $c = 5$ meters

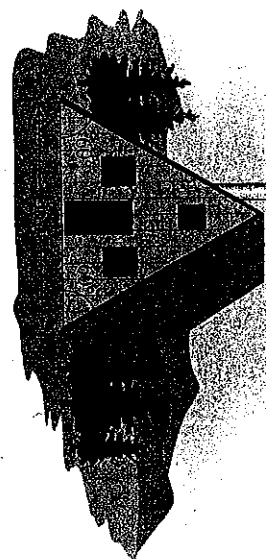
16. $a = 2$ meters, $b = 2$ meters, $c = 2$ meters

21. A commercial piece of real estate is priced at \$5.25 per square foot. Find the cost, to the nearest dollar, of a triangular lot measuring 260 feet by 320 feet by 450 feet.



20. The figure shows three roads that intersect to bound a triangular piece of land. Find the lengths of the other two sides of the land to the nearest foot.

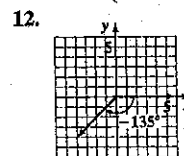
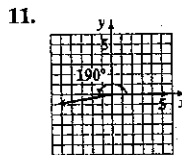
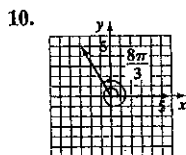
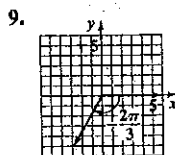
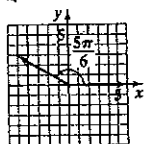
19. Two airplanes leave an airport at the same time on different runways. One flies on a bearing of N66.5°W at 325 miles per hour. The other airplane flies on a bearing of S26.5°W at 300 miles per hour. How far apart will the airplanes be after two hours?



17. The A-frame cabin shown below is 35 feet wide. The roof of the cabin makes a 60° angle with the cabin's base. Find the length of one side of the roof from its ground level to the peak. Round to the nearest tenth of a foot.

Lesson 1-3

- 4.5 radians 2. $\frac{\pi}{12}$ radians 3. $\frac{2\pi}{3}$ radians 4. $\frac{7\pi}{4}$ radians 5. 300° 6. 252° 7. -150°



- 40° 14. 275° 15. $\frac{5\pi}{4}$ 16. $\frac{7\pi}{6}$ 17. $\frac{4\pi}{3}$ 18. $\frac{15\pi}{2} \text{ ft} \approx 23.56 \text{ ft}$ 19. 20.6π radians per min 20. 42,412 ft per min

$$\sin t = -\frac{3}{5}, \cos t = -\frac{4}{5}, \tan t = \frac{3}{4}, \csc t = -\frac{5}{3}, \sec t = -\frac{5}{4}, \cot t = \frac{4}{3}$$

$$\sin t = -\frac{15}{17}, \cos t = \frac{8}{17}, \tan t = -\frac{15}{8}, \csc t = -\frac{17}{15}, \sec t = \frac{17}{8}, \cot t = -\frac{8}{15}$$

$$-\frac{2\sqrt{3}}{3} \quad 24. \sqrt{3} \quad 25. \text{undefined} \quad 26. \text{undefined} \quad 27. \cos t = \frac{\sqrt{21}}{7}, \tan t = \frac{2\sqrt{3}}{3}, \csc t = \frac{\sqrt{7}}{2}, \sec t = \frac{\sqrt{21}}{3}, \cot t = \frac{\sqrt{3}}{2} \quad 28. 1$$

$$1 \quad 30. -1 \quad 31. \sin \theta = \frac{5\sqrt{89}}{89}, \cos \theta = \frac{3\sqrt{89}}{89}, \tan \theta = \frac{5}{3}, \csc \theta = \frac{\sqrt{89}}{5}, \sec \theta = \frac{\sqrt{89}}{3}, \cot \theta = \frac{3}{5} \quad 32. \frac{7}{2} \quad 33. -\frac{1}{2} \quad 34. 1 \quad 35. 1$$

$$\cos 20^\circ \quad 37. \sin 0 \quad 38. 42 \text{ mm} \quad 39. 23 \text{ cm} \quad 40. 37 \text{ in.} \quad 41. \sqrt{15} \quad 42. 772 \text{ ft} \quad 43. 31 \text{ m} \quad 44. 56^\circ \quad 45. \sin \theta = -\frac{5\sqrt{26}}{26}$$

$$\theta = -\frac{\sqrt{26}}{26}, \tan \theta = 5, \csc \theta = -\frac{\sqrt{26}}{5}, \sec \theta = -\sqrt{26}, \cot \theta = \frac{1}{5} \quad 46. \sin \theta = -1; \cos \theta = 0; \tan \theta \text{ is undefined; } \csc \theta = -1, \sec \theta \text{ is undefined;}$$

$$\theta = 0 \quad 47. \text{quadrant I} \quad 48. \text{quadrant III} \quad 49. \sin \theta = -\frac{\sqrt{21}}{5}, \tan \theta = -\frac{\sqrt{21}}{2}, \csc \theta = -\frac{5\sqrt{21}}{21}, \sec \theta = \frac{5}{2}, \cot \theta = -\frac{2\sqrt{21}}{21}$$

$$\sin \theta = \frac{\sqrt{10}}{10}, \cos \theta = -\frac{3\sqrt{10}}{10}, \csc \theta = \sqrt{10}, \sec \theta = -\frac{\sqrt{10}}{3}, \cot \theta = -3 \quad 51. \sin \theta = -\frac{\sqrt{10}}{10}, \cos \theta = -\frac{3\sqrt{10}}{10}, \tan \theta = \frac{1}{3}, \csc \theta = -\sqrt{10},$$

$$\theta = -\frac{\sqrt{10}}{3} \quad 52. 85^\circ \quad 53. \frac{3\pi}{8} \quad 54. 50^\circ \quad 55. \frac{\pi}{6} \quad 56. \frac{\pi}{3} \quad 57. -\frac{\sqrt{3}}{2} \quad 58. -\sqrt{3} \quad 59. \sqrt{2} \quad 60. \frac{\sqrt{3}}{2} \quad 61. -\sqrt{3}$$

$$-\frac{2\sqrt{3}}{3} \quad 63. -\frac{\sqrt{3}}{2} \quad 64. \frac{\sqrt{2}}{2} \quad 65. 1 \quad 66. -\frac{\sqrt{3}}{2} \quad 67. \frac{\sqrt{3}}{2}$$

Lesson 4 →

94. $\frac{\pi}{2}$ 95. 0 96. $\frac{\pi}{4}$
 97. $-\frac{\pi}{3}$ 98. $\frac{2\pi}{3}$ 99. $-\frac{\pi}{6}$
 100. $\frac{\sqrt{2}}{2}$ 101. 1 102. $-\frac{\sqrt{3}}{3}$
 103. $-\frac{\sqrt{3}}{3}$ 104. 2 105. $\frac{4}{5}$ 106. $\frac{4}{5}$

107. $-\frac{3}{4}$ 108. $-\frac{3}{4}$ 109. $-\frac{\sqrt{10}}{10}$ 110. $\frac{\pi}{3}$ 111. $\frac{\pi}{3}$ 112. $-\frac{\pi}{6}$ 113. $\frac{2\sqrt{x^2+4}}{x^2+4}$ 114. $\frac{x\sqrt{x^2-1}}{x^2-1}$

Lesson 5

1. $C \approx 55^\circ, b \approx 10.5$, and $c \approx 10.5$ 2. $A = 43^\circ, a \approx 171.9$, and $b \approx 241.0$ 3. $b \approx 16.3, A \approx 72^\circ$, and $C \approx 42^\circ$
 4. $C \approx 98^\circ, A \approx 55^\circ$, and $B \approx 27^\circ$ 5. $C = 120^\circ, a \approx 45.0$, and $b \approx 33.2$ 6. two triangles; $B_1 \approx 55^\circ, C_1 \approx 86^\circ$, and $c_1 \approx 31.5$
 7. no triangle 8. $a \approx 59.0, B \approx 3^\circ$, and $C \approx 15^\circ$ 9. $B \approx 78^\circ, A \approx 39^\circ$, and $C \approx 63^\circ$
 10. $B \approx 25^\circ, C \approx 115^\circ$, and $c \approx 8.5$ 11. two triangles; $A_1 \approx 59^\circ, C_1 \approx 84^\circ, c_1 \approx 14.4; A_2 \approx 121^\circ, C_2 \approx 22^\circ, c_2 \approx 5.4$
 12. $B \approx 9^\circ, C \approx 148^\circ$, and $c \approx 73.6$ 13. 8 sq ft 14. 4 sq ft 15. 4 sq m 16. 2 sq m 17. 35 ft 18. 35.6 ml
 19. 861 mi 20. 404 ft; 551 ft 21. \$214,194